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On non-commutative open string theories

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Abstract: We investigate new compactifications of OM theory giving rise to a (3+1)-dimensional open string theory with non-commutative x^0-x^1 and x^2-x^3 coordinates. The theory can be directly obtained by starting with a D3-brane with parallel (near critical) electric and magnetic field components, in the presence of a RR scalar field. The magnetic parameter permits to interpolate continuously between the x^0-x^1 non-commutative open string theory and the x^2-x^3 spatial non-commutative U(N) super Yang-Mills theory. We discuss SL$(2,\mathbb{Z})$ transformations of this theory. Using the supergravity description of the large-N limit, we also compute corrections to the quark-antiquark Coulomb potential arising in the NCOS theory.

Keywords: D-branes, M-Theory, String Duality.

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Contents

1. Introduction

Recently there has been an increasing interest in non-commutative geometries [1, 2], where coordinates satisfy

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}\,. \tag{1.1}$$

It has been shown that theories on such spaces naturally arise in string theory as the worldvolume theory of Dp-branes in a $B_{\mu\nu}$-field background [2, 3, 4]. When $\theta^{0\mu} = 0$, then the low-energy effective theory on such branes is a non-commutative Super Yang-Mills (NCSYM) theory with 16 supercharges [2, 5]. In particular, starting with N coincident D3-branes along 0123 directions with a non-zero B_{23} field, one obtains a $\mathcal{N} = 4$, $D = 4$ non-commutative $\mathrm{U}(N)$ theory. The large-N limit of the NCSYM theories have been studied through the generalization of gravity/gauge theory correspondence to the cases with B-fields [6, 7, 8].

The case $\theta^{0\mu} \neq 0$ was recently studied in [9, 10], and it is relevant to the strong coupling limit of NCSYM theories. This point was investigated in [10], using the strong-weak coupling duality symmetry of type-IIB string theory, which transforms the B_{23} (magnetic) background field into a B_{01} (electric) background. As a result, the strongly coupled theory can be described in terms of a weakly coupled open string theory where the coordinate x^1 and the time coordinate x^0 are non commutative. The emergence of an open string theory — rather than a field theory — is not unexpected, since there are arguments indicating that a field theory in a non-commutative spacetime ($\theta_{0i} \neq 0$) cannot be unitary [11]. The resulting open string theory, which can be directly obtained by starting with a system of D3-branes in the presence of a near critical B_{01} field, has indeed a string scale α' which is of the same order of the non-commutative scale θ, so string effects cannot be disentangled from non-commutative effects. For a near critical electric field, one can show that closed string states all become very massive and decouple from the theory. One ends up with a consistent theory of open strings which does not require closed strings, non-commutative open string theory (NCOS) (for other recent works on spacetime noncommutativity see [12]–[15]).

A similar construction can be made for other Dp-branes, and also by starting with the M5-brane in a near critical three-form field strength. It is believed that the later leads to a theory of open membranes (OM theory) [16, 17], which is decoupled from eleven-dimensional gravity. One obtains in this way a web of open string/membrane theories connected by dualities, much like the web of dualities connecting string and M-theory compactifications [16]. These theories can thus be used to explore string phenomena without the complications of gravity.

The aim of this work is to explore another piece of this web by considering a D3-brane system in the presence of both B_{01} and B_{23} field components, which is equivalent to a configuration of parallel electric and magnetic vector fields on the brane (a discussion on electric and magnetic fields suggesting a new type of dualities involving Lorentz boosts can be found in [18]). We shall show that there is a limit (corresponding to a critical value of the B_{01} component) in which closed string modes are decoupled, leading to a family of non-commutative open string theories in x^0-x^1 and x^2-x^3 directions. When the non-commutative length scale in the x^2-x^3 direction is greater than the string scale α', for energies much lower than α', one obtains a NCSYM theory in x^2-x^3 directions, generically with a non-zero Θ term (i.e. of the form $\Theta \int \tilde{F}_{\mu\nu} F^{\mu\nu}$). We study these theories under the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of type-IIB superstring theory. For irrational Θ, a generic $\mathrm{SL}(2,\mathbb{Z})$ transformation maps our NCOS theory to another NCOS theory with different parameters (in particular, different non-commutative scales). For rational Θ, there is a (one parameter) family of $\mathrm{SL}(2,\mathbb{Z})$ rotations which map the NCOS theory to a NCSYM theory.

The paper is organized as follows. In section 2, we consider a D3-brane in presence of both electric and magnetic backgrounds, and determine the spectrum of open and closed strings. We show that there is a critical electric field limit for which the closed string modes decouple. Using the type-IIB supergravity solution corresponding to the D3-brane configuration with B_{01} and B_{23}, in section 3 we study the transformation properties of these general NCOS theories induced by the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of the type-IIB theory. Section 4 contains another derivation of these NCOS theories by compactifying OM theory along two arbitrary directions of a 2-plane. In section 5, we use the supergravity solutions which are the holographic dual of the NCOS theories to obtain a quark-antiquark potential in the NCOS theory at large N.

2. Open strings in the presence of B_{01} and B_{23}

Let us consider a D3-brane in the presence of a constant B-field with components B_{01} and B_{23}. Such constant B-field is equivalent to constant electric and magnetic vector fields on the brane pointing in the direction x^1. Open strings in constant gauge fields were originally discussed in [19]. Let x^μ, with $\mu, \nu = 0, 1, 2, 3$, label the directions that are parallel to the brane, and x^a, $a, b = 4, \ldots, 9$, the transverse directions. It is convenient to choose coordinates so that the closed string metric has the form $g_{\mu\nu} = \mathrm{diag}(-\zeta, \zeta, \eta, \eta)$, $g_{ab} = \delta_{ab}$, where ζ and η are constant parameters which will be fixed later. The boundary conditions for the open string coordinates are given by

$$\partial_\sigma X^0 + E\partial_\tau X^1 \Big|_{\sigma=0,\pi} = 0\,, \qquad \partial_\sigma X^1 + E\partial_\tau X^0 \Big|_{\sigma=0,\pi} = 0\,, \tag{2.1}$$

$$\partial_\sigma X^2 + B\partial_\tau X^3 \Big|_{\sigma=0,\pi} = 0\,, \qquad \partial_\sigma X^3 - B\partial_\tau X^2 \Big|_{\sigma=0,\pi} = 0\,, \tag{2.2}$$

$$E \equiv B^0{}_1 = -\zeta^{-1} B_{01}\,, \qquad B \equiv B^2{}_3 = \eta^{-1} B_{23}\,.$$

The transverse open string coordinates are free string coordinates obeying Dirichlet boundary conditions:

$$X^a \Big|_{\sigma=0,\pi} = x^a\,, \qquad a = 4, \ldots, 9\,,$$

where x^a denote the position in transverse space of N D3-branes. The solutions to the above boundary conditions are

$$\begin{aligned}
X^0 &= x^0 + 2\alpha'(p^0\tau - Ep^1\sigma) + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} e^{-in\tau} \left[ia_n^0 \cos(n\sigma) - Ea_n^1 \sin(n\sigma)\right], \\
X^1 &= x^1 + 2\alpha'(p^1\tau - Ep^0\sigma) + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} e^{-in\tau} \left[ia_n^1 \cos(n\sigma) - Ea_n^0 \sin(n\sigma)\right], \\
X^2 &= x^2 + 2\alpha'(p^2\tau - Bp^3\sigma) + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} e^{-in\tau} \left[ia_n^2 \cos(n\sigma) - Ba_n^3 \sin(n\sigma)\right], \\
X^3 &= x^3 + 2\alpha'(p^3\tau + Bp^2\sigma) + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} e^{-in\tau} \left[ia_n^3 \cos(n\sigma) + Ba_n^2 \sin(n\sigma)\right], \\
X^a &= x^a + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{\alpha_n^a}{n} e^{-in\tau} \sin(n\sigma)\,.
\end{aligned} \tag{2.3}$$

It is convenient to introduce the "open string parameters" $G_{\mu\nu}$, $\theta_{\mu\nu}$, G_s [2] as follows:

$$\begin{aligned}
G_{\mu\nu} &= g_{\mu\nu} - (Bg^{-1}B)_{\mu\nu} \\
&= \begin{pmatrix} -\zeta(1-E^2) & 0 & 0 & 0 \\ 0 & \zeta(1-E^2) & 0 & 0 \\ 0 & 0 & \eta(1+B^2) & 0 \\ 0 & 0 & 0 & \eta(1+B^2) \end{pmatrix},
\end{aligned} \tag{2.4}$$

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$$\theta^{\mu\nu} = 2\pi\alpha' \left(\frac{1}{g+B}\right)^{\mu\nu}_A = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\theta} \\ 0 & 0 & -\tilde{\theta} & 0 \end{pmatrix}, \tag{2.5}$$

$$G_s = g_s \left(\frac{\det G_{\mu\nu}}{\det(g_{\mu\nu}+B_{\mu\nu})}\right)^{1/2} = g_s\sqrt{(1-E^2)(1+B^2)}\,, \tag{2.6}$$

where

$$\theta = 2\pi\alpha' \frac{E}{\zeta(1-E^2)}\,, \qquad \tilde{\theta} = 2\pi\alpha' \frac{B}{\eta(1+B^2)}\,. \tag{2.7}$$

The canonical commutation relations for the string coordinates then imply the following commutation relations for the mode operators:

$$[a_n^\mu, a_m^\nu] = n\delta_{n+m}G^{\mu\nu}\,,$$

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}\,, \qquad [x^\mu, p^\nu] = iG^{\mu\nu}\,.$$

One can also check that the ends of the string do not commute, i.e.

$$[X^\mu(\tau,0), X^\nu(\tau,0)] = i\theta^{\mu\nu}\,, \qquad [X^\mu(\tau,\pi), X^\nu(\tau,\pi)] = -i\theta^{\mu\nu}\,.$$

Using (2.3), one can obtain the hamiltonian operator:

$$L_0 = \frac{1}{4}\alpha' G^{\mu\nu}p_\mu p_\nu + 2\sum_{n=1}^{\infty} G_{\mu\nu}a_{-n}^\mu a_n^\nu + 2\sum_{n=1}^{\infty}\alpha_{-n}^a\alpha_n^a - 1\,.$$

Now we set

$$\zeta = (1-E^2)^{-1}\,, \qquad \eta = (1+B^2)^{-1}\,, \tag{2.8}$$

so as to have

$$G_{\mu\nu} = \eta_{\mu\nu}\,, \qquad \theta = 2\pi\alpha' E\,, \qquad \tilde{\theta} = 2\pi\alpha' B\,. \tag{2.9}$$

The open string mass spectrum then becomes manifestly the same as the free string mass spectrum:

$$\alpha' M^2 = \alpha'(p_0^2 - p_1^2 - p_2^2 - p_3^2) = N - 1\,.$$

The addition of fermions is as in the usual free open superstring theory (with the appropriate change in the normal ordering constant). The closed string mass spectrum is then given by

$$\alpha'(1-E^2)(p_0^2 - p_1^2) - \alpha'(1+B^2)(p_2^2+p_3^2) = 2N + 2\bar{N} - 4\,.$$

We see that as $E \to 1$ the energy of the closed string states goes to infinity, while the energy of open string excitations remains finite. In the limit $E \to 1$ with fixed B, α' and G_s, the closed string states are thus decoupled from the theory. This is just as in the zero magnetic field case discussed in [9, 10]. Note that this limit requires $g_s \to \infty$ (see eq. (2.6)).

The resulting open string theory obtained in this limit contains the parameters G_s,

$$\theta = 2\pi\alpha' , \qquad \tilde{\theta} = 2\pi\alpha' B = \theta B . \tag{2.10}$$

In the next section it will be shown that one can also introduce another parameter χ associated with the RR scalar of type-IIB theory. So, altogether our NCOS theory is defined by four parameters, $\alpha', \tilde{\theta}, G_s$ and χ. The parameter $\alpha' = \theta/2\pi$ is the string scale and also characterizes the non-commutative scale in the x^0-x^1 directions, $\tilde{\theta}$ represents the non-commutative scale in the x^2-x^3 directions, G_s is the open string coupling, while χ is not relevant in the perturbative expansion.

General scattering amplitudes on the disc for this "non-commutative" open string theory will have the form

$$\begin{aligned} \left\langle V(p^1)\cdots V(p^N)\right\rangle_{\theta,\tilde{\theta}} = \exp\left[-\frac{i}{2}\sum_{n>m} p^n \wedge p^m \epsilon(\tau_n - \tau_m)\right] \times \\ \times \left\langle V(p^1)\dots V(p^N)\right\rangle_{\text{free string}} , \end{aligned} \tag{2.11}$$

where

$$p^n \wedge p^m = (p_0^n p_1^m - p_1^n p_0^m)\theta + (p_2^n p_3^m - p_3^n p_2^m)\tilde{\theta} .$$

In the case $B = 0$, one recovers the open strin theory of [9, 10], obtained from open strings in a purely electric background.

A special limit of this NCOS theory is given by $B \to \infty,\ \alpha' \to 0$, with

$$G_s = \text{fixed} , \qquad \alpha' B = \text{fixed} . \tag{2.12}$$

Because $\alpha' \to 0$, massive open string excitations also decouple in this limit, and one is left with the Super Yang-Mills field theory in non-commutative x^2-x^3 space. Thus the present family of NCOS theories interpolates between the purely electric NCOS theory of [9, 10] and the NCSYM.

In general, the theory contains two energy scales, given by θ and $\tilde{\theta}$, or α' and $B\alpha'$. At distances L much larger than $\sqrt{\theta}$, $\sqrt{\tilde{\theta}}$, the theory reduces to ordinary SYM theory. For $B \gg 1$, there is a regime $\theta \ll L^2 < \tilde{\theta}$ in which the theory is SYM field theory on the non-commutative space x^2-x^3; string effects can be ignored, but noncommutativity effects in x^2-x^3 directions are important. If B is of order 1 or lower, then string effects appear at the same time as non-commutative effects.

3. SL$(2, \mathbb{Z})$ transformations on NCOS theory

In this section we study the behavior of the open string theories of the previous section under type-IIB SL$(2, \mathbb{Z})$ symmetry. We start with the corresponding type-IIB supergravity solution. The lorentzian supergravity solution representing a D3-brane in the presence of B_{01} and B_{23} fields is given in [7] (see also [20]). In the string frame, it is

$$
\begin{aligned}
ds^2_{\rm str} &= f^{-1/2}\Big[h'(-dx_0{}^2+dx_1{}^2)+h(dx_2{}^2+dx_3{}^2)\Big]+f^{1/2}\Big[dr^2+r^2d\Omega_5^2\Big]\,, \qquad (3.1)\\
f &= 1+\frac{\alpha'^2R^4}{r^4}\,, \qquad h^{-1}=\sin^2\alpha f^{-1}+\cos^2\alpha\,,\\
h'^{-1} &= -\sinh^2\beta f^{-1}+\cosh^2\beta\,,\\
B_{01} &= -\tanh\beta f^{-1}h'\,, \qquad B_{23}=\tan\alpha f^{-1}h\,,\\
e^{2\phi} &= g_s{}^2hh'\,, \qquad \chi=\frac{1}{g_s}\sinh\beta\sin\alpha f^{-1}\ +\chi_0\,,\\
A_{01} &= \left(\frac{1}{g_s}\sin\alpha\cosh\beta+\chi_0\tanh\beta\right)h'f^{-1}\,,\\
A_{23} &= \left(\frac{1}{g_s}\sinh\beta\cos\alpha-\chi_0\tan\alpha\right)hf^{-1}\,,\\
F_{0123u} &= \frac{1}{g_s}\cos\alpha\cosh\beta\ hh'\partial_r f^{-1}\,.
\end{aligned}
$$

We have generalized the solution of [7] by adding an extra parameter χ_0 obtained by the $\mathrm{SL}(2,\mathbb{R})$ transformation $\chi\to\chi+\chi_0$.

In the $r\to\infty$ region, the metric asymptotically approaches the Minkowski metric, and the asymptotic values for the different fields are as follows:

$$
(B^\infty)^0{}_1=\tanh\beta\equiv E\,, \qquad (B^\infty)^2{}_3=\tan\alpha\equiv B\,, \tag{3.2}
$$

$$
\begin{aligned}
(A^\infty)^0{}_1 &= -\frac{1}{g_s}\frac{B}{\sqrt{1+B^2}}\frac{1}{\sqrt{1-E^2}}-\chi_0E\,,\\
(A^\infty)^2{}_3 &= \frac{1}{g_s}\frac{1}{\sqrt{1+B^2}}\frac{E}{\sqrt{1-E^2}}-\chi_0B\,, \qquad (3.3)
\end{aligned}
$$

$$
e^{2\phi_\infty}=g_s^2\,, \qquad \chi_\infty=\frac{1}{g_s}\frac{B}{\sqrt{1+B^2}}\frac{E}{\sqrt{1-E^2}}+\chi_0\,. \tag{3.4}
$$

We see that in the $E\to1$ limit, with $G_s,\theta,\tilde\theta$ and χ_0 fixed (see eqs. (2.6) and (2.10)), A^∞_{01} and A^∞_{23} also remain finite.

The effective open string dynamics is governed by the Born-Infeld action [21, 22]

$$
\begin{aligned}
S = &-\frac{1}{g_s(2\pi)^3(\alpha')^2}\int d^4x\sqrt{\det(g+B+2\pi\alpha'F)}\,+\\
&+\pi i\int\left(\chi\,\left(\frac{B}{2\pi\alpha'}+F\right)\wedge\left(\frac{B}{2\pi\alpha'}+F\right)+2(B+2\pi\alpha'F)\wedge A_2\right)+\\
&+\int C_4\,.
\end{aligned} \tag{3.5}
$$

The term involving χ gives rise to a Θ term in the low energy field theory, $\Theta=\chi_\infty$. It should be noted that there is an important difference between the NCOS limit at $\Theta = 0$ and the similar limit at $\Theta \neq 0$. While these NCOS theories with $\Theta \neq 0$ naturally arise by virtue of the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of type-IIB string theory, these theories have no independent perturbative expansion. It is not clear how much can be said about NCOS theories with Θ term. In particular, it is not clear how the analysis of Seiberg and Witten [2], which makes use of perturbative string theory, could be extended to this case.

The Born-Infeld action (3.5) is invariant under $\mathrm{SL}(2,\mathbb{Z})$ transformations [23] (in the low-energy limit this reduces to the Montonen-Olive duality of the $\mathcal{N} = 4$ SYM theory). The transformation of the fields can be inferred from the transformation properties under $\mathrm{SL}(2,\mathbb{Z})$ of the type-IIB supergravity fields [23].

Under the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of the type-IIB superstring the coupling

$$\lambda = \Theta + \frac{i}{g_s}\,, \qquad \Theta \equiv \chi_\infty\,,$$

transforms as

$$\lambda \longrightarrow \lambda' = \frac{a\lambda + b}{c\lambda + d}\,, \qquad ad - bc = 1\,, \tag{3.6}$$

where a, b, c, d form an $\mathrm{SL}(2,\mathbb{Z})$ matrix, whereas $B_{\mu\nu}$ and $A_{\mu\nu}$ (NSNS and RR) fields form a doublet:

$$\begin{pmatrix} B_{\mu\nu} \\ A_{\mu\nu} \end{pmatrix} \longrightarrow \begin{pmatrix} B'_{\mu\nu} \\ A'_{\mu\nu} \end{pmatrix} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} B_{\mu\nu} \\ A_{\mu\nu} \end{pmatrix}. \tag{3.7}$$

Therefore

$$g_s \longrightarrow g'_s = g_s |c\lambda + d|^2\,. \tag{3.8}$$

The Einstein metric $g^E_{\mu\nu} = e^{-\phi/2} g^{\mathrm{str}}_{\mu\nu}$ remains invariant, so the new string metric at $r = \infty$ is $|c\lambda + d|\eta_{\mu\nu}$. Using eqs. (3.3) and (3.7), one finds that the transformed electric and magnetic fields are

$$\begin{aligned} E \longrightarrow E' &= \frac{1}{|c\lambda + d|}\left[(d + c\Theta)E + c\frac{B}{G_s}(1 - E^2)\right], \\ B \longrightarrow B' &= \frac{1}{|c\lambda + d|}\left[(d + c\Theta)B - c\frac{E}{G_s}(1 + B^2)\right]. \end{aligned} \tag{3.9}$$

Let us now consider the $E \to 1$ limit for the SL(2) rotated parameters. In this limit the transformation (3.9) simplifies, since $g_s \to \infty$, $\lambda \to \Theta$. One can distinguish two different cases:

(a) Irrational Θ:

In this case there are no integers c and d such that $c\chi + d = 0$. In the $E \to 1$ and $g_s \to \infty$ limit, $|c\lambda + d|$ reduces to $|c\Theta + d|$ and E' and B' are:

$$\begin{aligned} E' &= \frac{1}{|c\Theta + d|}(d + c\Theta)E = \pm 1\,, \\ B' &= \pm B - \frac{c(1 + B^2)}{G_s|c\Theta + d|} = \text{finite}\,. \end{aligned} \tag{3.10}$$

In the $E \to 1$ limit, the electric and magnetic fields are obtained by the simple transformation rules

$$\frac{1-E^2}{|c\lambda+d|^2} \longrightarrow \frac{1-E'^2}{|c\Lambda+d|^2}\,, \qquad \frac{1+B^2}{|c\lambda+d|^2} \longrightarrow \frac{1+B'^2}{|c\Lambda+d|^2}\,,$$

where

$$\Lambda = \chi_0 + \frac{i}{G_s}\,, \qquad \chi_0 = \Theta - \frac{B}{G_s}\,. \tag{3.11}$$

The fact that $E'^2 \to 1$, with B' finite (and also $g'_s\sqrt{1-E'^2}$ remains finite) shows that an $\mathrm{SL}(2,\mathbb{Z})$ transformation leads to another NCOS with transformed parameters

$$(\theta, \tilde{\theta}, G_s, \Theta) \;\longrightarrow\; (\theta', \tilde{\theta}', G'_s, \Theta')\,,$$

where

$$\Theta' = \frac{a\Theta+b}{c\Theta+d}\,, \qquad G'_s = G_s|c\Lambda+d|^2\,. \tag{3.12}$$

To find $\theta'^{\mu\nu}$ and $G'_{\mu\nu}$ we use eqs. (2.4) and (2.5). One can choose coordinates (by scaling by suitable factors ζ and η given by (2.8) as in the previous section) so that the open string metric before the SL(2) transformation is $G_{\mu\nu} = \eta_{\mu\nu}$. We find

$$G'_{\mu\nu} = \frac{|c\Lambda+d|^2}{|c\lambda+d|}\,\eta_{\mu\nu}\,, \tag{3.13}$$

$$\theta' = 2\pi\alpha'\frac{E'(1-E^2)}{|c\lambda+d|(1-E'^2)} = 2\pi\alpha'\frac{|c\lambda+d|}{|c\Lambda+d|^2}\,, \tag{3.14}$$

$$\tilde{\theta}' = 2\pi\alpha'\frac{B'(1+B^2)}{|c\lambda+d|(1+B'^2)} = 2\pi\alpha' B'\frac{|c\lambda+d|}{|c\Lambda+d|^2}\,. \tag{3.15}$$

Rescaling the coordinates so that the new open string metric $G'_{\mu\nu}$ is equal to $\eta_{\mu\nu}$, we conclude that the new NCOS theory has parameters,

$$\theta' = 2\pi\alpha'\,, \qquad \tilde{\theta}' = 2\pi\alpha'\;B'\,. \tag{3.16}$$

(b) Rational Θ:

In this case there exists an $\mathrm{SL}(2,\mathbb{Z})$ transformation under which $c\Theta + d = 0$. The string coupling transforms as follows:

$$g_s \longrightarrow g'_s = g_s|c\lambda+d|^2 = \frac{c^2}{g_s}\,, \tag{3.17}$$

i.e. this transformation relates strong and weak coupling regimes. From (3.9) one can find transformed E and B in the $E \to 1$ limit:

$$E' = \pm\frac{B\sqrt{1-E^2}}{\sqrt{1+B^2}} \longrightarrow 0\,,$$

$$B' = \pm\frac{\sqrt{1+B^2}}{\sqrt{1-E^2}} \longrightarrow \pm\infty\,. \tag{3.18}$$

Therefore

$$\begin{aligned}\theta' &= 2\pi\alpha' \frac{E'(1-E^2)}{|c\lambda+d|(1-E'^2)} = 0\,,\\ \tilde{\theta}' &= 2\pi\alpha' \frac{B'(1+B^2)}{|c\lambda+d|(1+B'^2)} = 2\pi\alpha' \frac{G_s}{c} = \text{finite}\,,\end{aligned} \tag{3.19}$$

and the open string coupling is

$$G'_s = g'_s\sqrt{1-E'^2}\sqrt{1+B'^2} = g'_s B' = \frac{c^2(1+B^2)}{G_s} = \text{finite}\,. \tag{3.20}$$

In conclusion, for rational Θ there is an $\mathrm{SL}(2,\mathbb{Z})$ transformation which maps the NCOS theory to a NCSYM field theory, with $\Theta = a/c$. In particular, we see that whenever $\Theta = 0$ the NCOS theory is S-dual to NCSYM theory, even if both E and B are non vanishing. The reason is that in this case A_{01} given in (3.3) also vanishes, and therefore the S-dual theory will have a vanishing B_{01}. This generalizes the result of [10] that the theory with $E = 0,\ B \to \infty$ is S-dual to the theory with $E = 1,\ B = 0$. For irrational Θ, under $\mathrm{SL}(2,\mathbb{Z})$ transformations the NCOS theory is always transformed to a NCOS theory with new parameters given by the above transformation rules.

4. Six-dimensional origin

To clarify how these more general NCOS theories fit in the OM theory framework of [16], and also to exhibit the geometric origin of $\mathrm{SL}(2,\mathbb{Z})$ symmetry, it is useful to reproduce these theories by starting with OM theory. We shall first review OM-theory and its compactification on an electric circle [16, 17], and then study new compactifications on arbitrary directions of a 2-plane that lead to the general electric-magnetic NCOS theories obtained in section 2.

4.1 OM theory on an electric circle

We start with a configuration of M5 branes in the presence of a 3-form field strength. For later convenience, we choose the world-volume metric as follows:

$$g_{\mu\nu} = \mathrm{diag}(-\xi^2, \xi^2, \rho^2, \rho^2, \rho^2, \xi^2)\,. \tag{4.1}$$

The field strength is given by

$$H_{015} = -\xi^3 \tanh\beta\,, \qquad H_{234} = \rho^3 \sinh\beta\,. \tag{4.2}$$

The H field satisfies the non-linear self-duality constraints [24]. Note that H is rescaled by a factor of M_P^3 with respect to that of ref. [16], so H is dimensionless (M_P

is the eleven-dimensional gravitational scale). Now we take the limit $\beta \to \infty$ with

$$M_{\rm eff}^3 \equiv \frac{1}{2} M_P^3 \left(\xi^3 + H_{015}\right) = \frac{1}{2} M_P^3 \xi^3 (1 - \tanh\beta) = \text{fixed}\,, \tag{4.3}$$

$$M_P^3\, \xi\rho^2 = \text{fixed}\,. \tag{4.4}$$

The above quantities represent the effective tension (or energy per unit area) of an M2-brane lying on the 015 and 023 directions, respectively. This is achieved with

$$\xi = \xi_0 e^{2\beta/3}\,, \qquad \rho = \rho_0 e^{-\beta/3}\,. \tag{4.5}$$

with fixed ξ_0, ρ_0 and M_P (alternatively, one can send $M_P \to \infty$ and set $\xi = 1$, as done in [16]). As a result, H_{234} remains finite, and

$$M_{\rm eff} = M_P \xi_0\,.$$

Now let us consider the compactification of OM theory on a circle parametrized by the x^5 direction, $x^5 \equiv x^5 + 2\pi R_5$. One obtains N type-IIA D4-branes with a non-zero electric field along 01 directions, with the metric in the string frame given by

$$g_{\mu\nu}^{\rm str} = g_s^{2/3}\, \mathrm{diag}\left(-\xi^2, \xi^2, \rho^2, \rho^2, \rho^2\right), \tag{4.6}$$

and

$$g_s = (\xi R_5 M_P)^{3/2} = e^{\beta} (R_5 M_{\rm eff})^{3/2}\,, \qquad \alpha' = \frac{1}{M_{\rm eff}^3 R_5}\,, \tag{4.7}$$

$$B_{01} = M_P R_5\; H_{015} = -M_P R_5 \xi^3 \tanh\beta\,. \tag{4.8}$$

Thus

$$g_{\mu\nu}^{\rm str} = \xi_0 R_5 M_P\, \mathrm{diag}\left(-\xi_0^2 e^{2\beta}, \xi_0^2 e^{2\beta}, \rho_0^2, \rho_0^2, \rho_0^2\right), \tag{4.9}$$

$$E \equiv B^0{}_1 = \tanh\beta\,. \tag{4.10}$$

By setting

$$\xi_0 = (4 M_P R_5)^{-1/3}\,, \qquad \rho_0 = 2\xi_0\,,$$

the closed string metric becomes $g_{\mu\nu}^{\rm str} = \mathrm{diag}(-\frac{1}{4}e^{2\beta}, \frac{1}{4}e^{2\beta}, 1, 1, 1)$, and the open string metric is

$$G_{\mu\nu} = g_{\mu\nu}^{\rm str} - (B g_{\rm str}^{-1} B)_{\mu\nu} = \eta_{\mu\nu}\,.$$

Therefore the open string coupling G_s is given by

$$G_s = g_s \left(\frac{\det G_{\mu\nu}}{\det(g_{\mu\nu} + B_{\mu\nu})}\right)^{1/2} = g_s\sqrt{1 - E^2} = 2(M_{\rm eff} R_5)^{3/2} = \text{finite}\,. \tag{4.11}$$

Thus one obtains NCOS theory in $4+1$ dimensions. Compactifying along the x^4 direction one gets the $(3+1)$-dimensional NCOS theory with $B_{23} = 0$ (which is equivalent to compactifying OM theory on a 2-torus [16, 17]).

4.2 Oblique reduction of OM theory

Non-vanishing electric and magnetic fields can be generated by compactifying the theory in an oblique direction. It is convenient to choose coordinates so that the closed string metric is given by

$$g_{\mu\nu} = \mathrm{diag}\left(-\xi^2, \xi^2, \rho^2, \rho^2, \xi^2, \xi^2\right), \tag{4.12}$$

where the coordinates x_4 and x_5 are defined to be periodic with periods

$$x_5 \equiv x_5 + 2\pi R_5 , \qquad x_4 \equiv x_4 + 2\pi R_4^0 . \tag{4.13}$$

Note that this metric differs from the metric (4.1) of the previous subsection by a rescaling of the coordinate x_4. Now consider the M5-brane configuration of section 4.1, but with a rotation in the 2-plane x_4-x_5, i.e.

$$\begin{aligned} H_3 &= H_{01\tilde{5}}\ dx_0 \wedge dx_1 \wedge d\tilde{x}_5 + H_{23\tilde{4}}\ dx_2 \wedge dx_3 \wedge d\tilde{x}_4 , \\ \tilde{x}_4 &= x_4 \cos\alpha + x_5 \sin\alpha , \qquad \tilde{x}_5 = -x_4 \sin\alpha + x_5 \cos\alpha . \end{aligned} \tag{4.14}$$

The metric (4.12) remains invariant under this transformation. Then the H field components (which satisfy the self-duality conditions) are

$$H_{015} = -\xi^3 \tanh\beta \cos\alpha , \qquad H_{014} = \xi^3 \tanh\beta \sin\alpha , \tag{4.15}$$

$$H_{234} = \rho^2 \xi \sinh\beta \cos\alpha , \qquad H_{235} = \rho^2 \xi \sinh\beta \sin\alpha . \tag{4.16}$$

The theory should be decoupled from gravity in the limit $\beta \to \infty$ as above, with

$$\begin{aligned} M_{\rm eff}^3 = \frac{1}{2} M_P^3 \left(\xi^3 + H_{01\tilde{5}}\right) &= \frac{1}{2} M_P^3 \xi^3 \left(1 - \tanh\beta\right) = \text{fixed} , \\ M_P^3\ \xi\rho^2 &= \text{fixed} \end{aligned} \tag{4.17}$$

(equivalent conditions can be written, e.g. in terms of H_{015}). In addition, we also scale $\sin\alpha$ so that

$$\sinh\beta\ \ \sin\alpha = B = \text{fixed} . \tag{4.18}$$

The above conditions are satisfied by setting

$$\xi = \xi_0 e^{2\beta/3} , \qquad \rho = \rho_0 e^{-\beta/3} \tag{4.19}$$

with fixed ξ_0, ρ_0, B and M_P.

Dimensional reduction along x_5 gives as before N D4-branes with a B_{01} field, but now there is in addition a non-vanishing B_{23} component. Explicitly,

$$\begin{aligned} B_{01} &= M_P R_5\ H_{015} = -M_P R_5 \xi^3 \tanh\beta \cos\alpha , \\ B_{23} &= M_P R_5\ H_{235} = M_P R_5 \rho^2 \xi \sinh\beta \sin\alpha . \end{aligned} \tag{4.20}$$

The other components H_{014}, H_{234} lead to non-zero components of the 3-form RR field A_3. The metric is given by

$$g_{\mu\nu}^{\rm str} = g_s^{2/3}\, {\rm diag}\left(-\xi^2, \xi^2, \rho^2, \rho^2, \xi^2\right) \tag{4.21}$$

with g_s and α' given by eq. (4.7). The electric and magnetic field components are given by

$$B^0{}_1 = \tanh\beta\cos\alpha\,, \qquad B^2{}_3 = \sinh\beta\ \sin\alpha\,. \tag{4.22}$$

In the limit $\beta\to\infty$ we get $B^0{}_1 = 1$ and a magnetic field $B^2{}_3$ equal to B. Let us write $x_4 = R_4^0\theta_4$, where θ_4 is 2π periodic. In order to have a finite g_{44} component in the $\beta\to\infty$ limit, we set

$$R_4 = \frac{R_4^0}{\sqrt{1-(B^0{}_1)^2}} = \text{fixed}\,.$$

The compactification radius of the original (unrescaled) coordinate x_4 in the metric (4.1) is $\frac{\xi}{\rho}R_4^0 = \frac{\xi}{\rho}\sqrt{1-(B^0{}_1)^2}\, R_4$ and so remains finite in the $\beta\to\infty$ limit. Thus this is a compactification of OM theory on a finite torus in the natural OM frame [16] (i.e. the coordinate system in which the natural "open membrane" metric of the low energy field theory is finite).

By T-duality in the x_4 direction, one obtains N D3-branes, with non-vanishing NSNS and RR two-form components. After setting

$$\xi_0 = [4M_P R_5(1+B^2)]^{-1/3}\,, \qquad \rho_0 = 2\xi_0\,,$$

the closed string metric for the $(3+1)$-dimensional world-volume is

$$g_{\mu\nu} = {\rm diag}\left(-\frac{1}{1-E^2}, \frac{1}{1-E^2}, \frac{1}{1+B^2}, \frac{1}{1+B^2}\right),$$

and hence the open string metric and coupling become

$$G_{\mu\nu} = {\rm diag}(-1,1,1,1)\,, \tag{4.23}$$

$$G_s = g_s^B\sqrt{1-(B^0{}_1)^2}\sqrt{1+(B^2{}_3)^2} = 2\frac{R_5}{R_4}(1+B^2) = \text{finite}\,, \tag{4.24}$$

where we have used

$$g_{44} = \frac{R_4^2}{\alpha'}\,, \qquad g_s^B = \frac{g_s}{\sqrt{g_{44}}}\,.$$

In the $\beta\to\infty$ limit, the type-IIB gauge fields are as follows:

$$\begin{aligned} &B^0{}_1 = 1\,, \qquad B^2{}_3 = B\,, \qquad \chi = 0\,,\\ &A^0{}_1 = 0\,, \qquad A^2{}_3 = \frac{1}{G_s}(1+B^2)\,. \end{aligned} \tag{4.25}$$

This agrees with the asymptotic values of the gauge fields for the corresponding supergravity configuration (3.3) with $\chi_\infty = 0$ ($\chi_0 = -B/G_s$). In order to obtain the more general configuration with non-vanishing χ, one has to consider the reduction on a non-orthogonal torus x_4-x_5, characterized by a modular parameter $\tau = \tau_1 + i\tau_2$. [Equivalently, one can start with eq. (4.14) and introduce coordinates $(\tilde{x}_4, \tilde{x}_5)$ by a general non-orthogonal linear transformation in the (x_4, x_5) plane.] Then the metric has a non-vanishing non-diagonal component g_{45} which, upon dimensional reduction in x_5 and T-duality in x_4 gives rise to a non-zero χ field and also to a non-zero A_{01}, in accordance to eq. (3.3).

5. Supergravity description of large-N NCOS

The strong coupling $G_s \to \infty$ limit of U(N) NCSYM theory at fixed N leads to a NCOS theory with $B_{23} = 0$. Another interesting limit is the limit of large N with fixed 't Hooft coupling $2\pi G_s N = g^2_{\rm YM} N$. For large 't Hooft coupling, this regime can be studied by using a supergravity description, generalizing the dualities of [25]. The relevant supergravity background was described in [7], and it is obtained from (3.1) by scaling the parameters as follows:

$$\begin{aligned}
\cosh\beta &= \frac{\tilde{b}'}{\alpha'}\,, \qquad \cos\alpha = \text{fixed}\,,\\
x_{0,1} &= \frac{\tilde{b}'}{\sqrt{\alpha'}}\,\tilde{x}_{0,1}\,, \qquad x_{2,3} = \sqrt{\alpha'}\cos\alpha\;\tilde{x}_{2,3}\,,\\
r &= \sqrt{\alpha'}\;u\,, \qquad g = \frac{\tilde{b}'\cos\alpha}{\alpha'}\hat{g}\,,\\
R^4 &= \text{fixed} = 4\pi N\hat{g}\,, \qquad \chi_0 = \text{fixed}\,.
\end{aligned} \tag{5.1}$$

One obtains the following supergravity background [7]:

$$\begin{aligned}
ds^2 &= \alpha' f^{1/2}\left[\frac{u^4}{R^4}(-d\tilde{x}_0^2 + d\tilde{x}_1^2) + \frac{u^4}{R^4}\hat{h}(d\tilde{x}_2^2 + d\tilde{x}_3^2) + du^2 + u^2 d\Omega_5^2\right],\\
f &= 1 + \frac{R^4}{u^4}\,, \qquad \hat{h}^{-1} = 1 + \frac{u^4}{R^4\cos^2\alpha}\,,\\
B_{01} &= -\alpha'\frac{u^4}{R^4}\,, \qquad B_{23} = \alpha'\tan\alpha\frac{u^4}{R^4}\hat{h}\,,\\
e^{2\phi} &= \hat{g}^2 f^2\frac{u^8}{R^8}\hat{h}\,, \qquad \chi = \frac{1}{\hat{g}}\tan\alpha f^{-1} + \chi_0\,,\\
A_{01} &= \alpha'\frac{u^4}{R^4}\left(\frac{1}{\hat{g}}\tan\alpha + \chi_0\right), \qquad A_{23} = \alpha'\frac{u^4}{R^4}\hat{h}\left(\frac{1}{\hat{g}} - \chi_0\tan\alpha\right),\\
F_{0123u} &= \alpha'^2\frac{1}{\hat{g}}\hat{h}\frac{4u^3}{R^4}\,, \qquad R^4 = 4\pi N\hat{g}\,.
\end{aligned} \tag{5.2}$$

The coupling constant $\hat{g}$ can be identified with G_s. It is worth noting that $\tilde{x}^i, u, R$ are dimensionless. The only dimensionful parameter is α', which is fixed.

The case $\chi_0 = -\frac{1}{\hat{g}}\tan\alpha$ is special. In this case the $u = \infty$ value of χ is zero, and one has $A_{01} = 0$. As a result, the S-dual background (in which B_{01} and A_{01} are exchanged) has $B_{01} = 0$ and describes the purely magnetic case. This is in agreement with the observation of section 3 that the case $\Theta = 0$ is S-dual to NCSYM theory.

5.1 Calculation of Wilson loops

A "quark-antiquark" potential $V(L)$ for the large N $\mathcal{N} = 4$ SYM theory was computed in [26], with the result

$$V(L) = -\frac{c_0\sqrt{g_{\mathrm{YM}}^2 N}}{L}\,, \quad g_{\mathrm{YM}}^2 N = 2\pi\hat{g}N = \frac{1}{2}R^4\,, \quad c_0 = \frac{4\pi^2\sqrt{2}}{\Gamma(1/4)^4}\,. \tag{5.3}$$

The $1/L$ behavior is dictated by the conformal symmetry of the theory. The fact that $V(L)$ is proportional to $\sqrt{g_{\mathrm{YM}}^2 N}$ is believed to be a consequence of strong coupling physics. Non-commutative super Yang-Mills theory is not scale invariant, so the quark-antiquark potential $V(L)$ can have a more complicated dependence on L at distances which are comparable with the scale of noncommutativity. At large distances one should recover the potential (5.3) with the usual $-1/L$ behavior. A question of interest is how the noncommutativity of spacetime affects the forces between quarks and antiquarks at short distances. Are these forces weaker or stronger than the commutative case? At large N, this question can be addressed by using the supergravity description in terms of the background (5.2), and computing a Wilson line.

The case with $B_{01} = 0$ was already considered in [7], and it was found that strings cannot be localized near the boundary. In the case of $B_{01} \neq 0$, we will find that if the quark-antiquark pair lies on a plane orthogonal to the electric field direction, then the corresponding string configuration can indeed get to the boundary, and in this way gives rise to a quark-antiquark potential. For pairs which are aligned with the electric field (e.g. string configurations of the form $x_0 = \tau, x_1 = \sigma$) the string will stretch from the horizon to the boundary but will not connect, and hence will not give rise to a potential. This is expected: the electric field tends to separate the charges and for a near critical electric field it takes a lot of energy for the strings to bend and connect.

To illustrate how noncommutative effects modify the Coulomb potential at short distances, we consider a string configuration in this geometry of the form $x^0 = \tau$, $x^2 = \sigma$, $u/R \equiv U = \mathrm{U}(\sigma)$, $x_1 = x_3 = 0$ (representing a string lying on a plane which is orthogonal to the electric field). The Nambu-Goto action for this string in the background (5.2) is

$$S = \frac{T}{2\pi}\int d\sigma\sqrt{(1+U^4)\Big(U^4\hat{h} + R^2(\partial_\sigma U)^2\Big)}\,, \quad \hat{h} = \frac{1}{1+a^4U^4}\,, \; a^4 = \frac{1}{\cos^2\alpha}\,, \tag{5.4}$$

where the factor T arises from the integral over the time x_0.

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The solution to the equations of motion for U(σ) is given by

$$\frac{\sqrt{(1+U^4)}\; U^4\hat{h}}{\sqrt{U^4\hat{h}+R^2(\partial_\sigma U)^2}} = K = \text{const.} \tag{5.5}$$

Solving the above equation for $\partial_\sigma U$ we find

$$R^2(\partial_\sigma U)^2 = \frac{U^4}{K^2(1+a^4U^4)^2}\left[U^8+U^4-K^2a^4U^4-K^2\right]. \tag{5.6}$$

As a function of σ, U will vary from $U=\infty$ up to a minimum value U_0, given by the solution of $\partial_\sigma U=0$. In terms of U_0, the integration constant K is thus given by

$$K^2 = \frac{U_0^4(1+U_0^4)}{1+a^4U_0^4}. \tag{5.7}$$

The integration constant K (or, equivalently, U_0) can be determined by the condition that the quark separation is given by some distance L,

$$\begin{aligned} L(U_0) &= \int dx_2 = \frac{2Rb}{U_0}\int_1^\infty \frac{dy}{y^2}\frac{1+a^4U_0^4y^4}{\sqrt{(y^4-1)(U_0^4y^4+b^2)}}, \\ b &= \sqrt{\frac{1+U_0^4}{1+a^4U_0^4}}. \end{aligned} \tag{5.8}$$

The quark-antiquark interaction energy $\mathcal{E}$ then follows from the relation $S=T\mathcal{E}$. In this expression, one should subtract as in [26] the "quark masses" corresponding to the energy of two strings that stretch from the horizon to infinity. We obtain

$$\begin{aligned} V(U_0) = \mathcal{E}(U_0)-\mathcal{E}(0) &= \frac{RU_0}{\pi}\int_1^\infty dy\sqrt{1+U_0^4y^4}\left(\frac{y^2\sqrt{1+U_0^4y^4}}{\sqrt{(y^4-1)(U_0^4y^4+b^2)}}-1\right)- \\ &\quad -\frac{RU_0}{\pi}\int_0^1 dy\sqrt{1+U_0^4y^4}. \end{aligned} \tag{5.9}$$

Equations (5.8) and (5.9) define the quark-antiquark potential $V(L)$.

Let us first examine $V(L)$ in the case of large distances $L\gg 1$. Restoring α', this means $L\gg\sqrt{\alpha'}$, or $L^2\gg\theta$, where θ is the non-commutative scale parameter in the x_0-x_1 plane, given by $\theta=2\pi\alpha'$. As a function of U_0, $L(U_0)$ has a minimum value $L_{\min}$ at some $U_0=U_1$, and it goes to infinity in both the small and large U_0 regions (for small U_0 one has $L\sim 1/U_0$; for large U_0 one has $L\sim U_0$). Thus there are two possible values of U_0 for a given $L>L_{\min}$, and hence two possible values for the potential for a given L. It is easy to see that the branch $U_1<U_0<\infty$ has higher energy and it is unstable, so we restrict to the branch $0<U_0<U_1$. The large distance region is then obtained in the region of small U_0. To obtain the potential V as a function of L, we need to find $U_0=U_0(L)$ using (5.8). Although the integral (5.8) cannot

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be performed analytically, a systematic (large distance) expansion in powers of $1/L$ can be obtained by expanding the factor $1/\sqrt{y^4-1}$ in powers of $1/y$. Consider for example the purely electric case corresponding to $\alpha = 0$ (and hence $a = b = 1$). The resulting integrals are hypergeometric functions which can be expanded in powers of U_0. A given order in U_0 is given by a series which can be re-summed. We obtain the following result:

$$\begin{aligned} \frac{L}{R} &= \frac{a_0}{U_0} + a_1 U_0^2 + O(U_0^3)\,, \\ a_0 &= 2\int_1^\infty \frac{dy}{y^2}\frac{1}{\sqrt{y^4-1}} = \frac{2\sqrt{2}\pi^{3/2}}{\Gamma(1/4)^2}\,, \qquad a_1 = \frac{\Gamma(1/4)^2}{3\sqrt{\pi}}\,, \end{aligned} \tag{5.10}$$

or

$$U_0 = \frac{a_0 R}{L} + a_1 a_0^3 \frac{R^4}{L^4} + O\left(\frac{1}{L^5}\right). \tag{5.11}$$

Expanding the integral eq. (5.9) in powers of U_0 and inserting eq. (5.11) we obtain

$$\begin{aligned} V(L) &= -\frac{c_0\sqrt{g_{\mathrm{YM}}^2 N}}{L} - c_1\frac{(2g_{\mathrm{YM}}^2 N)^{5/4}}{L^4} + O(1/L^5)\,, \\ c_0 &= \frac{4\pi^2\sqrt{2}}{\Gamma(1/4)^4}\,, \qquad c_1 = \frac{16\pi^{9/2}}{3\Gamma(1/4)^6}\,. \end{aligned} \tag{5.12}$$

From this formula we can read the first corrections to a quark-antiquark potential due to the non-commutative nature of spacetime. Comparing this potential with the usual SYM case (represented by the first term in (5.12)), we see that the force between oppositely charged particles increases more rapidly as the distance is decreased from $L \gg \sqrt{\alpha'}$.

At shorter distances, the potential $V(L)$ is more complicated, but it can be computed numerically. Figure 1 shows a plot of $V(L)$ for the ordinary SYM theory and for the present non-commutative case. The curve $V(L)$ terminates at $L = L_{\min} = L(U_1)$ ($\cong 2.7$ in the figure). For shorter distances, there is no solution. Presumably, this may indicate a breakdown of the present simple description at very short distances. From the figure, we can see that the net effect of noncommutativity is increasing the force between a quark and an antiquark. A similar behavior is obtained for a generic $\alpha \neq \pi/2$, i.e. configurations with both electric and magnetic fields.

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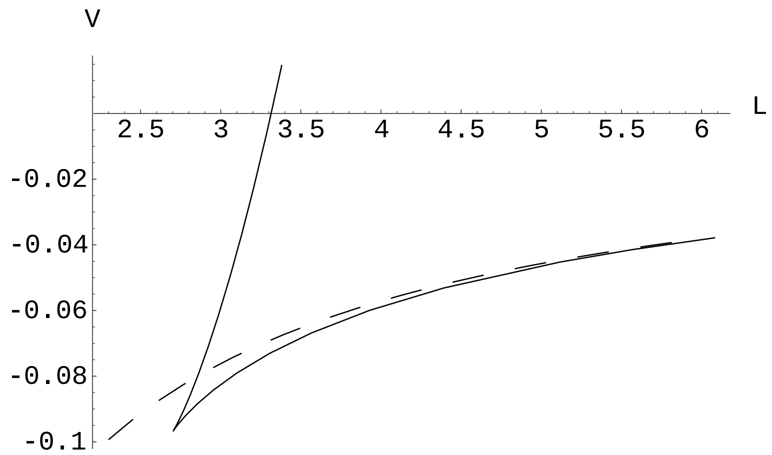

Figure 1: The quark-antiquark potential as a function of the distance L. The numerical plot is done for $R = 1$, $\alpha' = 1$, $a = 1$. The solid line crossing $V = 0$ is unphysical, describing the unstable branch. The dashed line represents the Coulombian potential $-c_0/L$ of the usual $\mathcal{N} = 4$ SYM theory.

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